ISSN 2347-3487

THE CONTRIBUTION OF DEGENERATE ELECTRON PRESSURE TO THE STABILITY OF THE OUTER REGION OF THIN KEPLERIAN ACCRETION DISKS AROUND A NEUTRON STAR

ABBI SEYOUM
ADVISOR: LEGESSE WETRO
ADDIS ABABA UNIVERSITY
ADDIS ABABA, ETHIOPIA

Abstract

The stability analysis of a geometrically thin, gas-pressure dominated accretion disk around a neutron star is presented. In purely radial perturbation case, thin disk is stable to thermal modes. The stability is analyzed at a small temperature, that is temperature approaching zero and at definite temperature. The contribution of both fully and partially degenerate electrons pressure for the stability of the disk in its outer region is investigated. We have found that the disk is stable in this region, where the gas pressure is more dominant than radiation pressure.

Council for Innovative Research
Peer Review Research Publishing System

Journal: JOURNAL OF ADVANCES IN PHYSICS
Vol. 11, No. 1
www.cirjap.com, japeditor@gmail.com

ISSN 2347-3487

Introduction

An accretion disk is a structure (often a circumstellar disk) formed by diffuse material in orbital motion around a central body. The central body is typically a younger star, a protostar, a white dwarf, a neutron star, or a black hole. The accretion disk is likely to be formed when the compact star is a member of a close binary system and matter transformed from a giant- type star on to its compact companion at high angular momentum. Following the pioneering works by Pringle and Rees (1972) [1], Shakura and Sunyaev (1973) [2], and Novikov and Thorne (1973) [3], a number of theoretical studies on the accretion disk has been made to account for observation of the X-ray sources. The importance of accretion is further manifested by the realization that probably a majority of all stars are members of binary systems which, at some stage of their evolution, undergo mass transfer. The study of interacting binary systems has revealed the importance of angular momentum in accretion. In many cases the transferred material cannot land on the accreting star until it has rid itself of most of its angular momentum. This leads to the formation of accretion disks, which turn out to be efficient machines for extracting gravitational potential energy and converting it in to radiation. There are two main reasons why many binaries transfer matter at some stage of their evolutionary life times [4]:

1. one of the stars in a binary separation shrinks, to the point where the gravitational pull of the companion can remove the outer layers of its envelope (Roche lobe overow);

2. one of the stars may, at some evolutionary phase, eject much of its mass in the form of a stellar wind, some of this material will be captured gravitationally by the companion (stellar wind accretion).

The stability of geometrically thin accretion disk has been studied extensively and it has been found that the disk is thermally and viscously unstable if it is optically thick and radiation pressure dominated (Pringle, Rees and Pacholczyk, 1973; Lightman and Eardley, 1976) [5]. There is also a possible mode of pulsational over stability. In this case, one looks for instability in which oscillation on the orbital time scale grow in amplitude because of the effects of viscosity (Lin and Paploizou, 1996) [6]. Kato (1978) [7] considered the evolution of all three components of the fluid velocity. He found that the disk exits pulsation instability besides the viscous instability and thermal instability. Chan and Tamm (1995) [8] pointed out the galactic black hole candidates may be due to pulsational overstability. Some early analysis about the stability of gas pressure dominated disc have incoperated azimuthal perturbation (livo and shaviv 1977, 1981; van horn; wesemal, and wingert 1980) [9]. However, the radial perturbation was neglected in all those studies. Mckee (1991) [10] has investigated the contribution of gas pressure to the stability of a standard disk. He found that the disk is stable when $\beta \le 0.6$ (where β is the ratio of radiation pressure to total pressure). This implies gas pressure dominated disk is more stable.

Electron Degeneracy Pressure

In order to estimate the electron pressure, we have to take account of the Pauli Exclusion Principle, which postulates that no two electrons can occupy the same quantum state, that is, have the same momentum and the same spin. Since an electron can have two spin states (up and down), this means that each element of phase space (location and momentum space) can be occupied by two electrons at most. The pressure generated by electrons that are forced in to higher momentum states as their density increases is called electron degeneracy pressure [11]. A state of complete degeneracy is obtained when all the available momentum states are occupied up to a maximum momentum value. Such an ideal situation can only be achieved at absolute zero temperature.

The force provided by this pressure sets a limit on the extent to which matter can be squeezed together without it collapsing into a neutron star.

The density of electrons is described by Fermi-Dirac statistics since an electron has half- integral spin. For an electron with momentum p the density in the range (dp) can be described by [12].

$$n_e(p)dp = \frac{8\pi}{h^3}p^2dp\left[\exp\left(\alpha + \frac{E}{K_B T_c}\right) + 1\right]^{-1} \ldots\ldots (1)$$

From Pauli Exclusion Principle, two identical electrons can't occupy the same state, that is

$$P(p)dp = \left[\exp\left(\alpha + \frac{E}{K_B T_c}\right) + 1\right]^{-1} \ldots\ldots (2)$$

which cannot be greater than 1.

Non-relativistic Complete Degeneracy

We consider a simple case, where T approaches to zero. When the density is high enough, all the electron states with energy less than a maximum energy are filled. Applying the Heisenberg and Pauli principle to a completely degenerate isotropic electron gas yields the momentum distribution; the number of electrons with momenta in the interval of $(p,\ p + dp)$ per unit volume:

$$n_e(p) = \frac{2}{\Delta V} = \frac{2}{h^3}4\pi p^2 dp \ldots\ldots (3)$$

Where p_o is the maximum momentum and $p \le p_o$. The maximal momentum p_o can be obtained by integration of $n_e(p)$:

$$p_o = \left(\frac{3h^3 n_e}{8\pi}\right)^{\frac{1}{3}} \ldots\ldots (4)$$

Using the pressure integral for the electron degenerate pressure:

ISSN 2347-3487

$$P = \frac{1}{3}\int_0^{p_o} vpn_e(p)dp \ldots\ldots (5)$$

where is P Pressure, v particles velocity and p is the momentum.

The electron degenerate pressure becomes

$$P_e = \frac{8\pi}{15mh^3{p_o}^5} \ldots\ldots (6)$$

$$P_e = \frac{h^2}{20m{m_p}^{5/3}}\left(\frac{3}{\pi}\right)^{2/3}\left(\frac{\rho_c}{\mu_e}\right)^{5/3} = 9.77\times 10^6\left(\frac{\rho_c}{\mu_e}\right)^{5/3} \ldots\ldots (7)$$

where m is mass of electron, μ_e is the ratio of electron number to proton number and ρ_c is central density of accretion disk.

For the radial structure of the accretion disk and first assuming that the equation of state of an ideal gas:

$$P_g = \frac{\rho_c K_B T_c}{m_p \mu} \ldots\ldots (8)$$

Where T_c is the central temperature, μ is the mean molecular weight, and m_p is the mass of proton.

The equation of accretion disk pressure with the effect of electron degenerate pressure can be written using equations (7) and (8) as

$$P = \frac{\rho_c K_B T_c}{m_p \mu} + 9.77\times 10^6\left(\frac{\rho_c}{\mu_e}\right)^{5/3} \ldots\ldots (9)$$

For an axisymmetric magnetized disk around a neutron star having a dipole moment aligned with its rotation axis in steady state, the half-thickness of the accretion disk can be calculated as:

$$H = (GM)^{-1/2}\left[\frac{\rho_c K_B T_c}{m_p \mu} + 9.77\times 10^6\left(\frac{{\rho_c}^2}{{\mu_e}^5}\right)^{1/3}\right]^{1/2} r^{3/2} \ldots\ldots (10)$$

where r radius of accretion disc.

Temperature in the Outer Region of the Disk

The central temperature in the outer part of the accretion disk can be calculated using a formula:

$$T_c = (2.7\times 10^7 K)(N+1)^{-1/20}\tilde{I}^{-1/5}\,{\alpha_{SS}}^{-1/5}(2\mu)^{1/4}\left(\frac{3r_g}{r_o}\right)^{3/4}$$

$$\times\left(\frac{M}{M_\odot}\right)^{-1/2}(\dot{M}_{14})^{3/10}\left(\frac{r}{r_o}\right)^{-3/10} f^{3/10} \ldots\ldots (11)$$

Moreover, $I = \frac{(2^N N!)^2}{(2N+1)!}$, $\tilde{I} = \frac{3}{2}I(N+1)$, $r_g = \frac{2GM}{C^2}$, $f = 1-\left(\frac{r_o}{r}\right)^{1/2}$

In which c is the speed of light, $r_o = r_A$ is Alfvens radius and N = 3, for electron scattering.

Constants	Values
α_{ss}	0.01
μ	0.62
$M/M_\odot$	1.4
$r_o = r_A$	$1\times 10^6 m$
μ_e	1
$M_\odot$	$1.99\times 10^{30} kg$
K_B	$1.38\times 10^{-23} m^2 kgs^{-2} k^{-1}$
h	$6.63\times 10^{-34} m^2 kgs^{-1}$
m_e	$9.11\times 10^{-31} kg$
$m_p = m_H$	$1.63\times 10^{-23} kg$

Table: Some basic constants with their corresponding values.

Substituting all the constants and writing the absolute central temperature in terms of the radius of the accretion disk, r gives:

$$T_c = 5.57 \times 10^{8.45} r^{-9/10} \left[r^{1/2} - 10^3\right]^{3/10} \text{K} \qquad (12)$$

The more simplified central density as a function of radius, r in the outer region of the accretion disk where gas-pressure is dominant is given by the equation

$$\rho_c = (5.3)\frac{(N+1)^{-17/40}}{\bar{I}^{7/10}}\alpha^{-7/10}(2\mu)^{9/8}\left(\frac{3r_g}{r_o}\right)^{15/8}$$
$$\times\left(\frac{M}{M_\odot}\right)^{-5/4}\left(\dot{M}_{14}\right)^{11/20}\left(\frac{r}{r_o}\right)^{-15/8} f^{11/20} \qquad (13)$$

This is simplified to be

$$\rho_c = 7.21 \times 10^{8.7} r^{-43/20} \left[r^{1/2} - 10^3\right]^{11/20} \text{kg/m}^3 \qquad (14)$$

Stability of the Accretion Disk

Suppose we start the disk in stationary state at the mean accretion rate. If this state is perturbed by small temperature increase, α goes up, and by the increased viscous stress the mass flux $\dot{M}$ increase. This increases the disk temperature further, resulting in a runaway to a hot state. Since $\dot{M}$ is larger than the average, the disk empties partly, reducing the surface density and the central temperature. A cooling front then transforms the disk to a cool state with an accretion rate below the mean. The disk in this model switches back and forth between hot and cool states.

The contribution of electron degenerate pressure to the stability of accretion disk in the outer region of the disk is calculated from stability relation [19]

$$\beta = \frac{P_g}{P_g + P_e} \qquad (15)$$

where P_g is gas pressure and P_e is electron degenerate pressure.

For gas-pressure dominated regions the disk is stable if

$$\bar{\beta} < \frac{3}{5} \qquad (16)$$

Where $\bar{\beta} = 1-\beta$

Stability in the Outer Region

The stability of accretion disk in the outer region due to electron degeneracy and gas pressure is

$$\beta < \frac{2}{5} \qquad (17)$$

The gas pressure in the region is given by

$$P_g = 5.34 \times 10^{22.15} r^{-3.05} \left[r^{1/2} - 10^3\right]^{17/20} \qquad (18)$$

The Complete electron degenerate pressure is given by

$$P_e = 2.63 \times 10^{22.5} r^{-3.58} \left[r^{1/2} - 10^3\right]^{11/12} \qquad (19)$$

The Partial electron degenerate pressure is given by

$$P_e = 3.29 \times 10^{3.95} r^{-3.05} \left[r^{1/2} - 10^3\right]^{1/20} \qquad (20)$$

Substituting equations (18) and (19) in to equation (17) helps to get the radius of accretion disk. That is

$$3.2 \times 10^{22.15} r^{-3.05} \left[r^{1/2} - 10^3\right]^{17/20} - 1.05 \times 10^{22.5} r^{-3.58} \left[r^{1/2} - 10^3\right]^{11/12} < 0 \qquad (21)$$

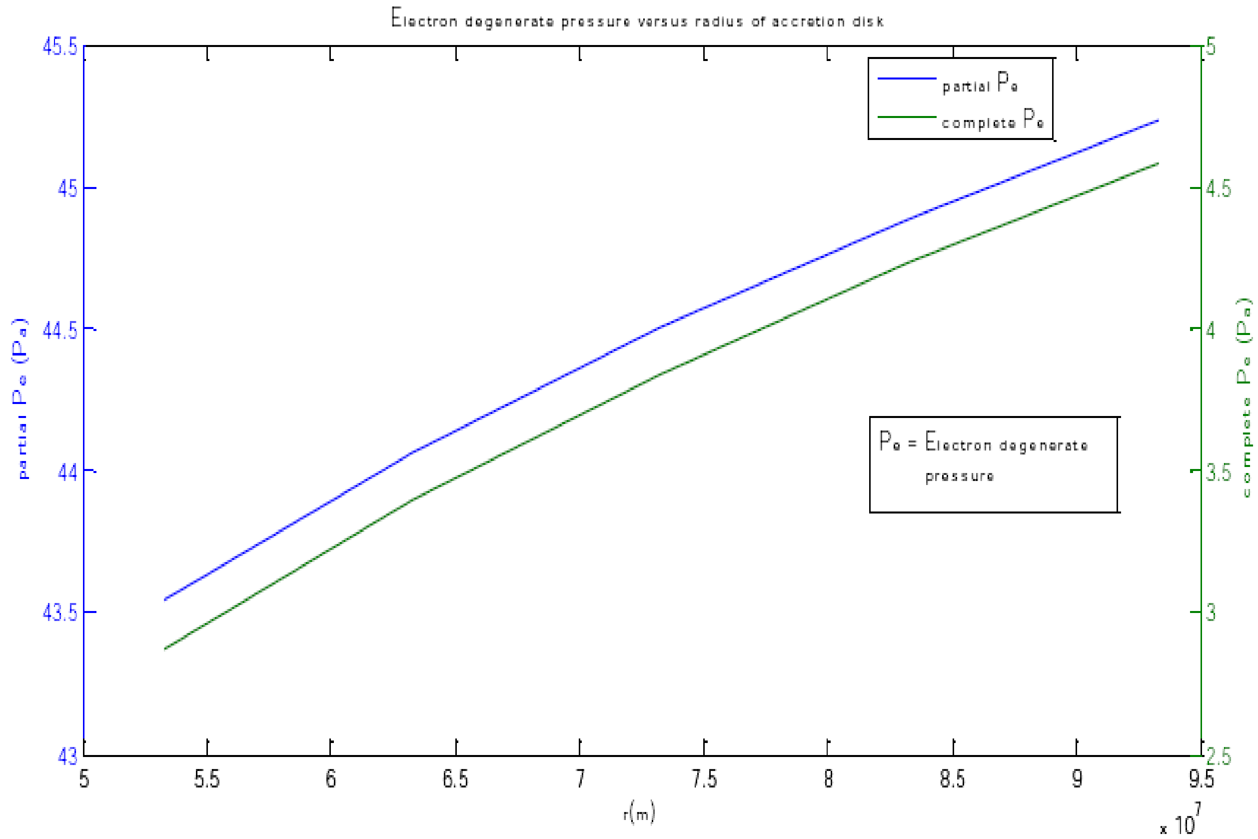
If the accretion disk is stable, then its radius in the outer region is approximately between $5.33 \times 10^7\text{m}$ and $1 \times 10^8\text{m}$. Calculating the value of r from equation (21) yields

$$5.33 \times 10^7\text{m} \leq r \leq 1 \times 10^8\text{m} \qquad (22)$$

Similarly, substituting equations (18) and (20) in to equation (17) and find the value of r, we see similar relation as equation (22).

The following figure shows that as a radius of an accretion disk around a neutron star increases both complete and partial electron degenerate pressure increases in the outer part of the accretion disk.

Moreover, as the radius of the accretion disk increases, partial electron degenerate pressure increases more than the complete electron degenerate pressure.



Conclusion

Investigating the contribution of both fully and partially degenerate electrons pressure for the stability of thin keplerian accretion disks around a neutron star in its outer region where both the gas and electron degenerate pressure are dominant, we have found that the disk is stable. Therefore, the disk is stable in the outer region though the electron degenerate pressure contributes to the total pressure.

Bibliography

[1] Pringle, J.E., and Rees, M.J. Astron. Astrophysics., 21, 1.

[2] Shakura N.I., and Sunyaev R.A., 1973, A.A, 24, 337.

[3] Novikov, ID., and Thorne, K.S. 1973, in Black Hole, ed.c. DeWitt and B. DeWITT (Gordon and Breach), New York.

[4] J. Frank, A. King and D. Raine. Accretion Power in Astrophysics. Third Edition, Combrige University Press.

[5] Pringle, J.E., Rees, M.J. and Pacholczyk, A.G.: 1973, Astron. Astrophys. 29, 179.

[6] Lin and Blumenthal. 1996, Astrophys. Space Sci. 322.

[7] Kato, S.: 1978, Mon. Not. R.Astron. Soc. 185, 629.

[8] Chen, X. and Tamm, R.E,: 1995, Astrophysics. J. 412, 254.

[9] R. Kippenhahn and A. Weingert.: 1990, Stellar Structure and Evolution, Springer-Verlag.

[10] Mckee, M.R.: 1991, Astron. Astrophysics. 251, 689.

[11] D. Prialnik. An Introduction to the Theory of Stellar Structure and Evolution. Combridge University Press

[12] K. Huang. Stastical Mechanics. [New York: Wily, 1987], p. 247.

[13] J. McDougall and E.C.Stoner, “The computation of Fermi-Drac Functions,” Phil.Trans. Roy. London 237 (1939) 67.

[14] John David Jackson, classical electrodynamics, 3rd edn, John Wiley and Sons, Inc. 2001.

[15] Rees, M.J. 1972, Astro, Astrophysics.21, 1.

[16] A. Odrzywolek et al. “Detection Possibility of the pair-annimation neutrinos form the neutrino-cooled pre-supernova star”, astro-ph/0311012.

ISSN 2347-3487

[17] Galions. An Introduction to Galios Theory. p, 426.

[18] M. Capitelli, CM.Ferreira, B.F Gordiets, A.I.Osipov. Atomic, Optical, and Plasma Physics. Bari, Lisbon, Mosco, March, 2000.

[19] N. Straumann. General Relativity and Relativistic Astrophysics. Springer-verlong. Berlin Heidelberg. New York, Tokyo, 1984.

[20] S.T Petcov, “On the Non-Adiabatic Neutrino Oscillation in Matter,” B. 1987, 299.

[21] The Student Edition of Matlab. The Ultimate Computing Environment for Technical Education. Version 4, New Jersy, Mathworks inc, 1995.